# Locally most powerful group-sequential tests when the groups are formed randomly

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**1.** Let a sequential statistical experiment be conducted in the following way: at each stage, we observe a group of random variables $(X_{i,1}, X_{i,2}, \dots, X_{i,n_i})$; the observations $X_{i,j}$ are i.i.d. with the distribution $P_\theta$, $\theta \in \Theta$, where $\Theta$ is an open subset of the real line, and the size of the $i$-th group, $n_i$, is defined by a random variable $\nu_i$. Let $\kappa_n$, a non-decreasing function of $n$, be the cost to obtain a group of $n$ observations (in the simplest case $\kappa_n \equiv 1$). We consider a problem of testing a simple hypothesis $H_0 : \theta = \theta_0$ against a composite alternative $H_1 : \theta > \theta_0$, where $\theta_0 \in \Theta$ is some fixed point.

The goal is to construct a locally most powerful (LMP) test for this problem, i.e. a test maximizing the slope of the power function at $\theta = \theta_0$ in the class of all sequential tests such that type I error and the average overall cost of the experiment do not exceed the given constants.

**2.** Let $f_\theta$ be the probability density function (a Radon-Nikodym derivative) of $P_\theta$, with respect to some $\sigma$-finite measure $\mu$, for all $\theta \in \Theta$.

Suppose the fulfillment of the following conditions:

C1. $\exists \gamma_1 : \limsup_{\theta \to \theta_0} I(\theta_0, \theta)/(\theta - \theta_0)^2 = \gamma_1 < \infty$,
where $I(\theta_0, \theta) = E_{\theta_0} \ln f_{\theta_0}(X_1)/f_\theta(X_1)$ is the Kullback-Leibler information.

C2. $\exists\ \dot{f}_{\theta_0}$: $\dot{f}_{\theta_0}$ is integrable (with respect to $\mu$) and

$$\int |f_\theta - f_{\theta_0} - (\theta - \theta_0)\dot{f}_{\theta_0}| d\mu = o(\theta - \theta_0),$$

as $\theta \to \theta_0$, i.e. $\dot{f}_{\theta_0}$ is the Fréchet derivative of $f_\theta$ at $\theta = \theta_0$ in $L_1(\mu)$.

C3. $\exists \delta_1 < \infty : E\nu_i < \delta_1\ \forall i \in \mathbb{N}$.

C4. $\exists \delta_2, \delta_3 : 0 < \delta_2 \le E\kappa_{\nu_i} \le \delta_3 < \infty\ \forall i \in \mathbb{N}$.

**3.** Then the LMP group-sequential test is defined by the stopping time

$$\tau = \inf \left\{ k : \sum_{i=1}^{k} \sum_{j=1}^{n_i} \frac{\dot{f}_{\theta_0}(X_{i,j})}{f_{\theta_0}(X_{i,j})} \notin (-A, B) \right\}$$

and the terminal decision to reject $H_0$ if

$$\sum_{i=1}^{k} \sum_{j=1}^{n_i} \dot{f}_{\theta_0}(X_{i,j})/f_{\theta_0}(X_{i,j}) > B,$$

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where $A$ and $B$ are some positive numbers.

**4.** In case $P(\nu_i = 1) = 1$ for all $i \geq 1$, the LMP group-sequential test becomes a LMP sequential test considered in [1], [2], [3]. An optimal group-sequential test for testing a simple hypothsis against a simple alternative for discrete distributions was considered in [4].

## References

[1] R.H. Berk (1975). Locally most powerful sequential tests. *Annals of Statistics* 3: 373–381.

[2] A. Novikov, P. Novikov (2010). Locally most powerful sequential tests of a simple hypothesis vs. one-sided alternatives. *Journal of Statistical Planning and Inference* 140(3): 750–765.

[3] A. Novikov, P. Novikov (2011). Locally most powerful sequential tests of a simple hypothesis vs. one-sided alternatives for independent observations. *Theory Probab. Appl.* 56(3)

[4] X.I. Popoca Jiménez (2012). *Optimilidad de pruebas de hipótisis secuenciales con grupos de tamaño aleatorio.* Master Thesis, UAM – Iztapalapa, Mexico City, Mexico.

[5] A. Novikov, P. Novikov. Locally most powerful group-sequential tests when the groups are formed randomly. *To be submitted.*