Optimal trade execution and price manipulation in order books with time-varying liquidity

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1. *Summary:* In financial markets liquidity is not constant over time but exhibits strong seasonal patterns. We consider a limit order book model that allows for time-dependent, deterministic depth and resilience of the book and determine optimal portfolio liquidation strategies. In a first model variant we propose a trading dependent spread that increases when market orders are matched against the order book. In this model no price manipulation occurs and the optimal strategy is of the "wait region – buy region" type often encountered in singular control problems. In a second model we assume that there is no spread in the order book. Under this assumption we find that price manipulation can occur, depending on the model parameters. Even in the absence of classical price manipulation there may be transaction-triggered price manipulation. In specific cases, we can state optimal strategies in closed form. The talk is based on [8].

2. Empirical investigations have demonstrated that liquidity varies over time. In particular deterministic time-of-day and day-of-week liquidity patterns have been found in most markets. In spite of these findings the academic literature on optimal trade execution usually assumes constant liquidity during the trading time horizon. In the talk we relax this assumption and analyze the effects of deterministically varying liquidity on optimal trade execution for a risk-neutral investor. We characterize optimal strategies in terms of a trade region and a wait region and find that optimal trading strategies depend on the expected pattern of time-dependent liquidity. In the case of extreme changes in liquidity, it can even be optimal to entirely refrain from trading in periods of low liquidity. Incorporating such patterns in trade execution models can hence lower transaction costs.

Time-dependent liquidity can potentially lead to price manipulation. In periods of low liquidity, a trader could buy the asset and push market prices up significantly; in a subsequent period of higher liquidity, he might be able to unwind this long position without depressing market prices to their original level, leaving the trader with a profit after such a round trip trade. In reality such round trip trades are often not profitable due to the bid-ask spread: once the trader starts buying the asset in large quantities, the spread widens, resulting in a large cost for the trader when unwinding the position. We propose a model with trading-dependent spread and demonstrate that price manipulation does not exist in this model in spite of time-dependent liquidity. In a similar model with fixed zero spread we find that price manipulation or transaction-triggered price manipulation (a term recently

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coined by [2] and [9]) can be a consequence of time-dependent liquidity.

Our liquidity model is based on the limit order book market model of [11], which models both depth and resilience of the order book explicitly. The instantaneously available liquidity in the order book is described by the depth. Market orders issued by the large investor are matched with this liquidity, which increases the spread. Over time, incoming limit orders replenish the order book and reduce the spread; the speed of this process is determined by the resilience. We generalize the model of [11] in that both depth and resilience can be time dependent. In relation to the problem of optimal trade execution we show that there is a time dependent optimal ratio of remaining order size to bid-ask spread: If the actual ratio is larger than the optimal ratio, then the trader is in the "trade region" and it is optimal to reduce the ratio by executing a part of the total order. If the actual ratio is smaller than the optimal ratio, then the trader is in the "wait region" and it is optimal to wait for the spread to be reduced by future incoming limit orders before continuing to trade. We will see that allowing for time-varying liquidity parameters brings qualitatively new phenomena into the picture. For instance, it can happen that it is optimal to wait regardless of how big the remaining position is, while this cannot happen in the framework of [11].

Building on empirical investigations of the market impact of large transactions, a number of theoretical models of illiquid markets have emerged. One part of these market microstructure models focuses on the underlying mechanisms for illiquidity effects, e.g., [10] and [7]. We follow a second line that takes the liquidity effects as given and derives optimal trading strategies within such a stylized model market. Two broad types of market models have been proposed for this purpose. First, several models assume an instantaneous price impact, e.g., [5], [4] and [3]. The instantaneous price impact typically combines depth and resilience of the market into one stylized quantity. Time-dependent liquidity in this setting then leads to executing the constant liquidity strategy in volume time or liquidity time, and no qualitatively new features occur. In a second group of models resilience is finite and depth and resilience are separately modelled, e.g., [6], [11], [1] and [12]. Our model falls into this last group. Allowing for time-dependent depth and resilience leads to higher technical complexity, but allows us to capture a wider range of real world phenomena.

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